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Exploring the richness of center-surround dynamics: A bifurcation study

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Abstract

The balance of excitatory and inhibitory interactions between neurons is one of the characteristic aspects of neural computation. In both neural network and neural field models these interactions have been modeled using center-surround connectivity kernels. Depending on the relative strength of excitation and inhibition these networks have been found to exhibit rich and interesting dynamical behavior. Although many models have been reported in the literature using center-surround connectivity kernels and many experimental studies have shown evidence for changes in observed behavior from winner-take-all to gain control, a thorough bifurcation analysis of these networks in terms of sensitivity of the network to peak strength, discriminability of the peaks and speed of convergence has not been done. In our present work we visit this question in order to identify the parameter regimes where this important switch in the behavior of the network occurs and also establish the trade offs that arise with the choice of a particular connectivity kernel.

Keywords: Center-surround interactions, lateral inhibition, winner take all, neural fields, bifurcation analysis

Introduction

▶ Context

Lateral inhibition leading to competition among neurons has been found to produce a number of different behaviors (e. g., winner take all, oscillations)

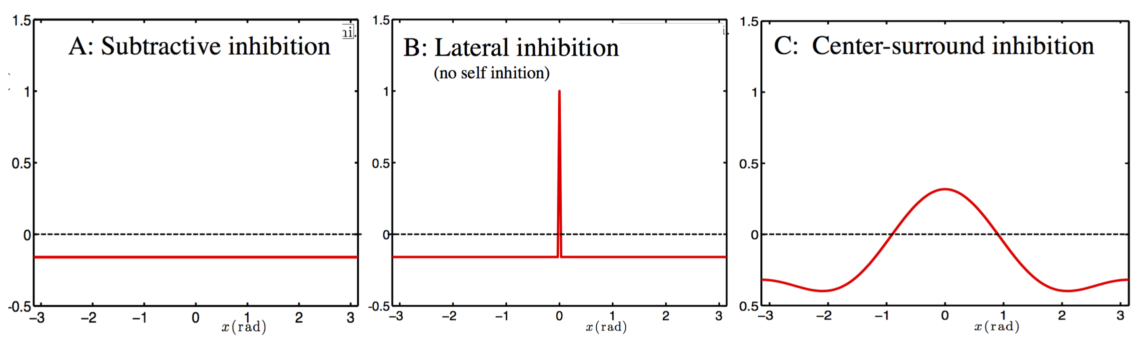
This kind of competition has been explored in a number of tasks such as:

- Short term memory in hippocampus [1].
- Velocity estimation in MT [3].
- Contrast enhancement in retinal cells [6].

▶ State of the Art

Recurrent neural networks with lateral inhibition have been studied in many ways.

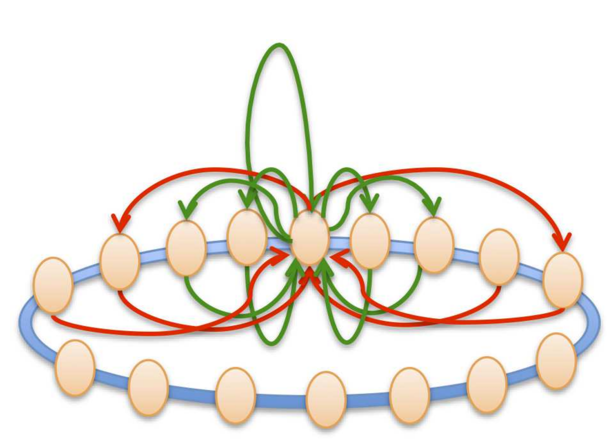
- Variety of assumptions:
  - Task (e.g. short term memory, single maximum).
  - Activation function (e.g. sigmoid, step, ramp).
  - Dynamics of local inhibition (e.g. instantaneous, slow).
  - Synaptic weights of lateral inhibition.



- Variety of results
  - Conditions for existence and uniqueness of a WTA solution [2].
  - Strong lateral inhibition and no self inhibition give WTA [3].
  - Lateral inhibition along with self inhibition leads to gain regularization [3].
  - Center-surround connectivity could exhibit multi stable behavior [4].
  - Impact of truncated connectivity was shown [5].

▶ Goal

We are interested in understanding the behavior of a group of recurrently connected network of neurons. Considering a ring model and using neural fields formalism we focus on **center-surround** connectivity kernels.



- We analyze sensitivity of the neural field to the peak strength and discriminability to the peak separation using **bifurcation analysis**.
- Specific questions:
  - **Is it possible to study various connectivity kernels using a general method?**
  - In which conditions does the network exhibit a winner take all behavior?
  - In which conditions does the network exhibit multi-stability?
  - What is the impact of surround excitation?

Model Description

▶ Ring model of orientation selection

The activity of a population of neurons is denoted by $u(x,t)$ with a feature space of orientation $x \in [-\pi, \pi)$.

The neural field equation is given by:

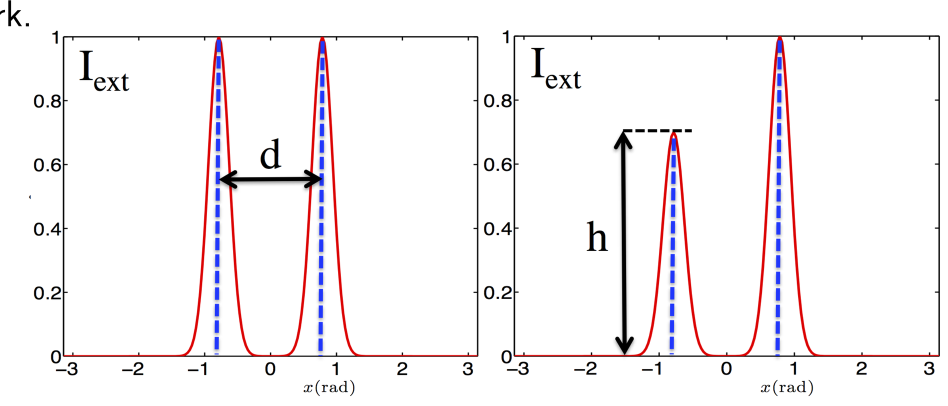
$$\frac{du(x,t)}{dt} = -u(x,t) + \int_{-\pi}^{\pi} J(x,y) S(\mu u(y,t)) dy + I_{ext}(x),$$

where,

- J is the connectivity kernel in direction space x,
- S is a sigmoid function with slope μ,
- I_{ext} is the input.

▶ Definition of I_{ext}

I_{ext} is defined as a two bumps input parameterized by peak separation (d) and amplitude on one peak (h) allowing us to study the discriminability of the network.



▶ Definition of J

J is defined as a weighted difference of gaussians:

$$J(x) = g_e G(x, \sigma_e) - g_i G(x, \sigma_i),$$

where, $G(x,\sigma)$ is a one dimensional Gaussian function $\left(G(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\frac{-x^2}{2\sigma^2}\right)$.

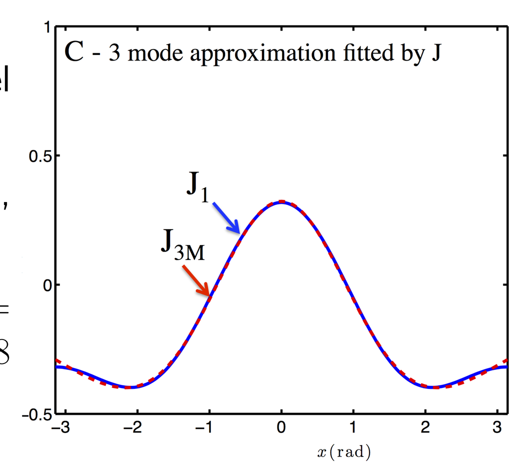
The four parameters of J $(g_e, \sigma_e, g_i, \sigma_i)$ allow us to describe kernels introduced earlier:

- Connectivity A: $g_e \to o, g_i > 0, \sigma_i >> 2\pi$.
- Connectivity B: $g_e > 1, \sigma_i \to 0, g_i > 0, \sigma_i >> 2\pi$.
- Connectivity C: $g_e > g_i, \sigma_e < \sigma_i$.

Considering the three mode kernel explored by [4],

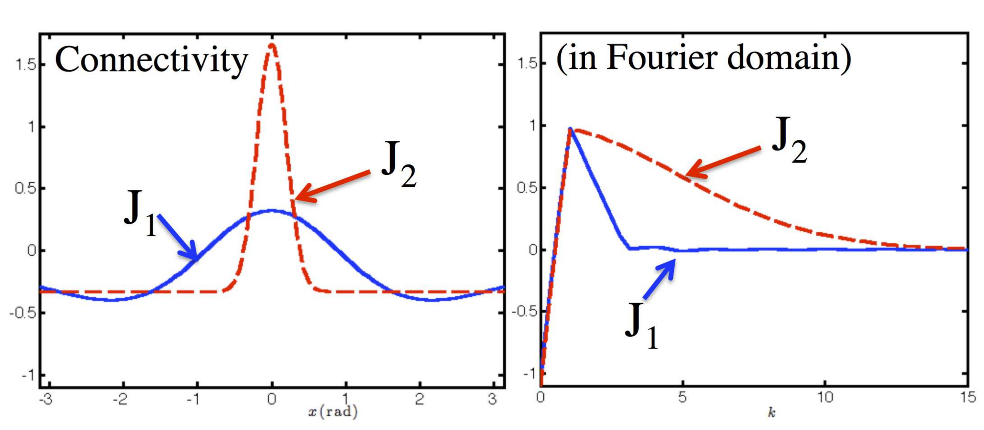
$J_{3M}(x) = J_0 + 2J_1 \cos x + 2J_2 \cos(2x)$, where J_0=-1, J_1=1 and J_2=1/2,

best fit of J is obtained for $\sigma_e = 0.97, w_e = 2.65, \sigma_i = 2.27, w_i = 4.38$ denoted by J_1.

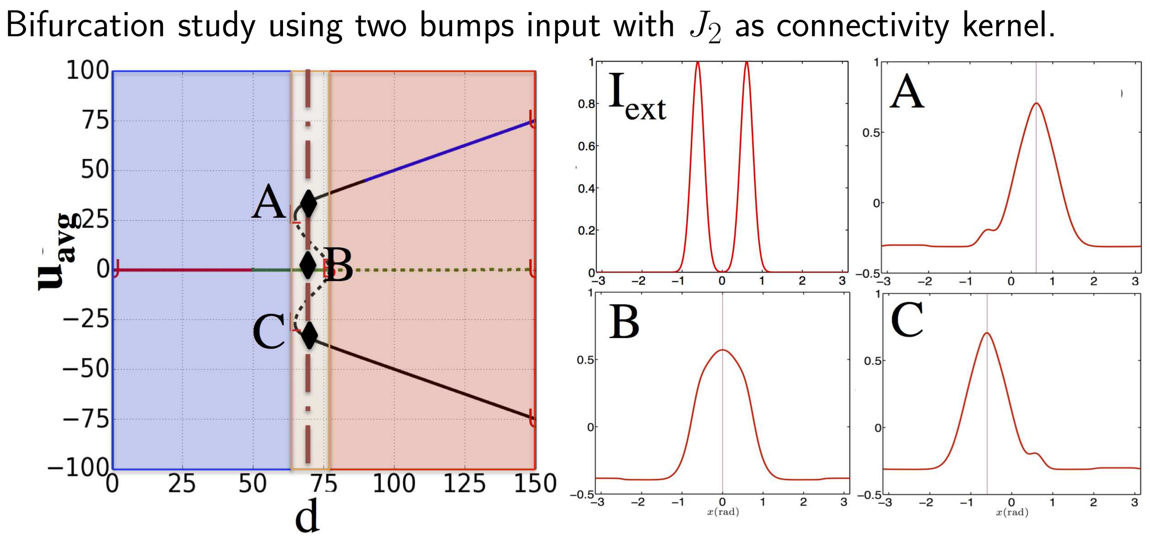


Bifurcation Study

We begin our analysis by considering a variant of J_1, J_2 having different excitatory and inhibitory regions but matching first modes in the Fourier spectra.

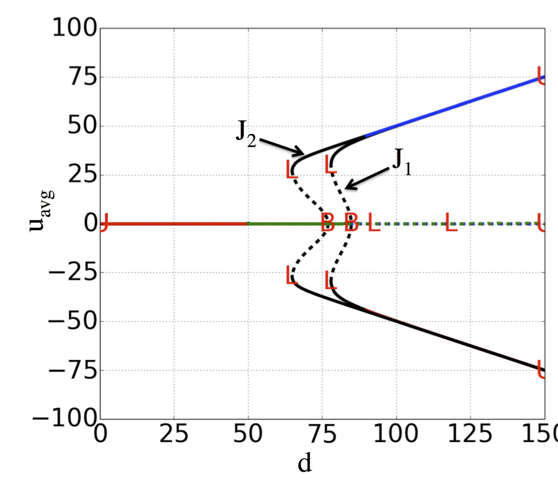


Bifurcation study using two bumps input with J_2 as connectivity kernel.



Bifurcation Study

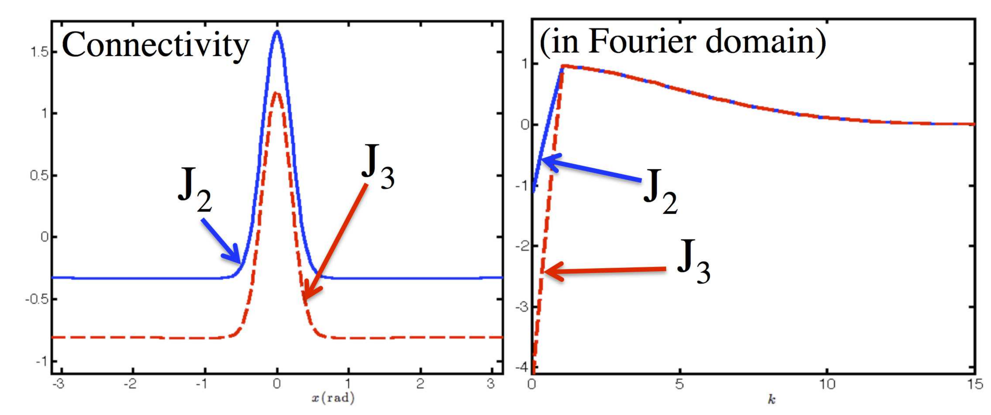
▶ Bifurcation maps for J_1 and J_2 connectivity



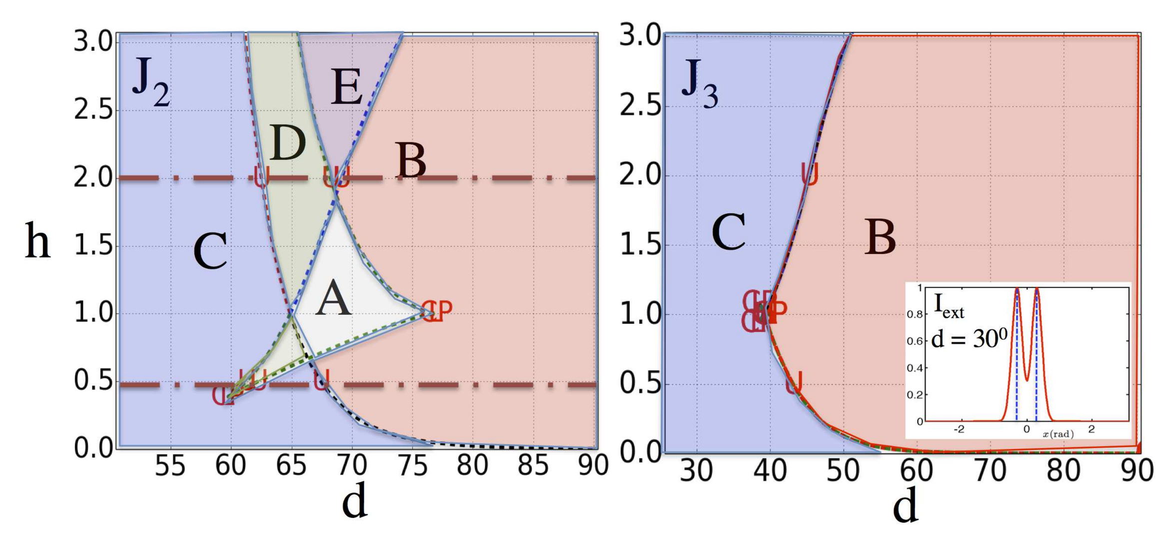
- The bifurcation maps are qualitatively the same.
- Peak discriminability improves with increased lateral inhibition.

▶ Impact of surround excitation

To investigate the effect of increased surround inhibition, we consider a variant of J_2, J_3 by adding a negative value.

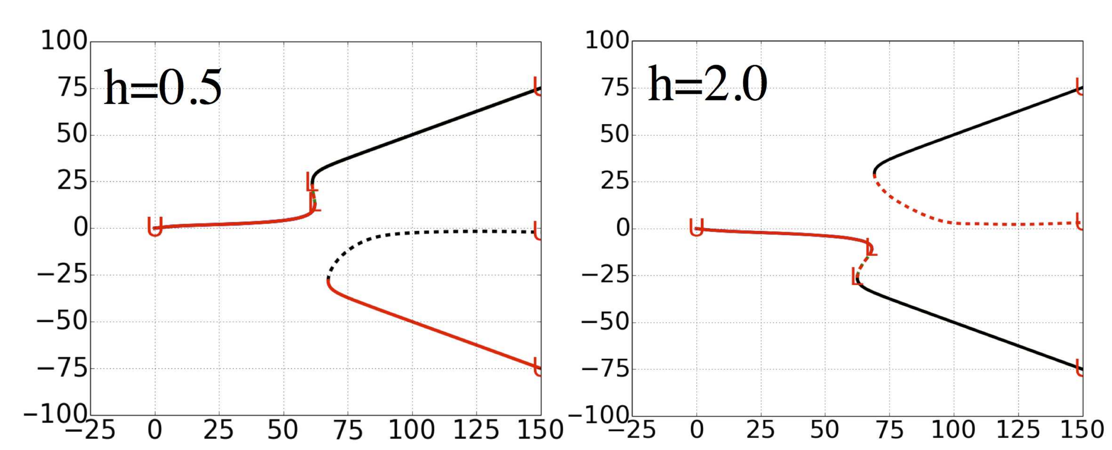


▶ Bifurcation study (d and h)



A: Tri-stability, B: Bi-stability between two bumps, C: Average
D: Bi-stability between dominant peak and average, E: Winner (h)

One parameter continuation w.r.t. d at h=0.5, 2.0



Conclusion

- We presented a **general** method for studying various connectivity kernels that have been explored in the literature.
- When surround excitation is considered, the peak activity of the network may not be aligned with the peak activity in the input.
- Network exhibits peak tuning and high discriminability for purely inhibitory kernel.

▶ Future work

- Bifurcation study incorporating uniform inhibition kernel and local kernels allowing for multiple winners.
- Implications of surround excitation for models of perceptual multistability.

References

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